Résumé du projet de recherche (Langue 2)

This study of structured and sparse system. We plan to exploit this structure and propose a new algorithm for the general problem of decomposing a symmetric tensor. Isolation is of crucial importance for the team in many applications. If time permits, we will also focus on these algorithms. We have obtained specific results for one type of structure: hence we obtain dedicated algorithm and sharp complexity results by reducing the problem of solving bilinear systems to determinantal ideals. We also propose ad hoc techniques to handle symmetries. In order to have a uniform approach of these problems, we introduce sparse Grabner bases, an analog of classical Groebner bases for semi group algebras, and 11c propose sparse 1 ariants of the classical F4 / F_5 and FGLM algorithms to compute them. The PhD project will focus on the following two directions. Sparse Grabner bases We have proposed variants of F_5 and FGLM to solve symbolically systems whose support lie in the same support (unmixed systems); we have also obtained sharp complexity results when the monomials in the support are integer points in a lattice polytope. We want to estimate two new research directions. First, a possible extension of this work would be the generalization to mixed systems (where the algorithms and the complexity would depend on the Newton polytopes of each of the monomials in turn of the system). A first goal of the current research is to indicate that such a generalization may be possible. Another restriction of the result, is that the complexity analysis (bounding the maximal degree occurring in the Groebner basis computation) is for the moment restricted to the polytopal case. However, a classical question is to bound the number of solutions in the algebraic closure of a polynomial system where all polynomials have generic coefficients. When the exponent vectors of the monomials are points with integer coordinates in a lattice polytope, Kushnirenko’s theorem shows that the number of solutions is bounded by the normalized volume of the polytope. A natural question that arises is then to extend the work we have done in the polytopal case to the case where only a small subset of monomials appear in the equations (jiewnomial systems). A noticeable result would be to compute a sparse Grabner basis of such a system in polynomial time when the number of monomials in the support is close to the number of variables. From an implementation point of view, we feel that implementing efficiently this new generation of algorithms in a general framework could be a new research area. Probably algorithms from con ex geometry or the combinatorial world would be necessary components of an efficient implementation. Also, merging our existing approach (the so called sparse-Matrix F5) with a Buchberger’s type approach 1 could lead to a termination criterion of the algorithm in the non-regular cases and for positive dimensional systems. 3 Algorithms for symmetric tensor decomposition - the real case A symmetric tensor is a higher order generalization of a symmetric matrix. The decomposition of a symmetric tensor consists into writing it using a minimal number, called symmetric rank, of linear combinations of symmetric outer products of vectors. This could be seen as a generalization of cigenrnluc decomposition of symmetric matrices to higher order symmetric tensors. When the rank is small there are algorithms for computing it efficiently. However, neither their theoretical nor their practical performance is well understood. Every symmetric tensor is equivalent to a homogeneous polynomial, where the dimension of the tensor corresponds to the degree and its order corresponds to the number of variables. Therefore, the problem of symmetric tensor decomposition is equivalent to write a homogeneous polynomial as a sum of powers of linear forms. A first goal is to consider optimal algorithms for decomposing the tensor over the complex numbers A relevant question is to generalize the study to decompose symmetric tensors m-er the real numbers. We will also study the problem of decomposition algorithms over the real numbers. Such algorithms will strongly depend on real root isolation algorithms of univariate polynomial. Since real root isolation is of crucial importance for the team in many applications, if time permits, we will also focus on these algorithms. We would like to mention that when the rank is big the problem of decomposition could be reduced to a highly structured polynomial system. We plan to exploit this structure and propose a new algorithm for the general problem of decomposing a symmetric tensor. This study of structured and sparse
Another restriction of the result, is that the complexity analysis (bounding the maximal degree occurring in the Grobner basis computation) is for the moment restricted to the polytopal case. However, a classical question is to bound the number of solutions in the algebraic closure of a polynomial system where all polynomials have generic coefficients. When the exponent vectors of the monomials are the points with integer coordinates in a lattice polytope, Kushnirenko’s theorem shows that the number of solutions is bounded by the normalized volume of the polytope. A natural question that arises is then to extend the work we have done in the polytopal case to the case where only a small subset of monomials appear in the equations (polynomial systems). A noticeable result would be to compute a sparse Grobner basis of such a system in polynomial time when the number of monomials in the support is close to the number of variables. From an implementation point of view, we feel that implementing efficiently this new generation of algorithms in a general framework could be a new research area. Probably algorithms from complex geometry or the combinatorial world would be necessary components of an efficient implementation. Also, merging our existing approach (the so-called sparse-Matrix F5) with a Buchberger's type approach 1 could lead to a termination criterion of the algorithm in the non-regular cases and for positive dimensional systems.